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THE FUNDING GAME: PERFORMANCE INCENTIVES FOR LOCAL TRANSPORT PLANS

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Abstract

This paper examines the recently introduced performance-based funding system for English local transport authorities, using game theory to help analyse: how the overall structure of the system incentivises the players; how the detailed features of the system affect its incentive properties and its effectiveness; and what alternative incentive systems might be considered by a government department wishing to achieve the best possible performance at local level.

The researchers' initial task was to understand and document the real game as it is currently played, and this was done with the assistance of interviews with key individuals in the Department for Transport and English local authorities (Kelly *et al,* 2006). Next, game theory models were reviewed for relevance/applicability, and two possible analogues were found for the Local Transport Plan game: the rank order tournament of Lazear and Rosen (1981) or the rent-seeking contest (Tullock, 1980). Ultimately, Clark and Riis' (1998) multi-player, multi-prize model of the rent-seeking contest was adopted and the game was solved for the 85 player Local Transport Plan case. Using the results, an analysis is given of the current LTP game, including its overall structure and specific features.

Our findings point to the prize structure (how many prizes and how 'graduated') as a key issue in the design of optimal incentives in this application. The existence of a 'level playing field' for competition between different local authorities is another issue which has a large bearing on the optimal incentive structure – and which DfT has given attention to in practice. Finally, we assess alternative possible incentive structures in the paper, including their possible application to the LTP game and the conditions under which the game theory work suggests they are most likely to be successful.

1. Aims, objectives and method

The aim of this paper is to provide an analysis of the recently introduced performance-based funding system for English local transport authorities.

Specifically, the paper attempts to use game theory[1] to help understand:

- how the overall structure of the system incentivises the players;
- how the detailed features of the system affect its incentive properties and its effectiveness;
- what alternative incentive systems might be considered by a government department wishing to achieve the best possible performance at local level.

The method adopted in this work included the following steps:

- understand and document the real game as it is currently played, using interviews with key individuals in the English local authorities and Department for Transport, and a postal survey (Kelly *et al*, 2006);
- consider alternative game theory models and evaluate for relevance/applicability;
- develop a theoretical model based on the LTP game, returning to discuss challenging elements of the model with the real players;
- solve the model, investigate sensitivity to parameters, and draw out implications for incentive system design.

The paper is structured as follows: Section 2 gives a brief background to the Local Transport Plans; Section 3 describes the performance-based funding system, from a game theoretic perspective; Section 4 introduces and develops a game theoretic model, and presents the findings; finally Section 5 draws conclusions on the work and makes proposals for further research, some of which will be conducted over the next 12 months.

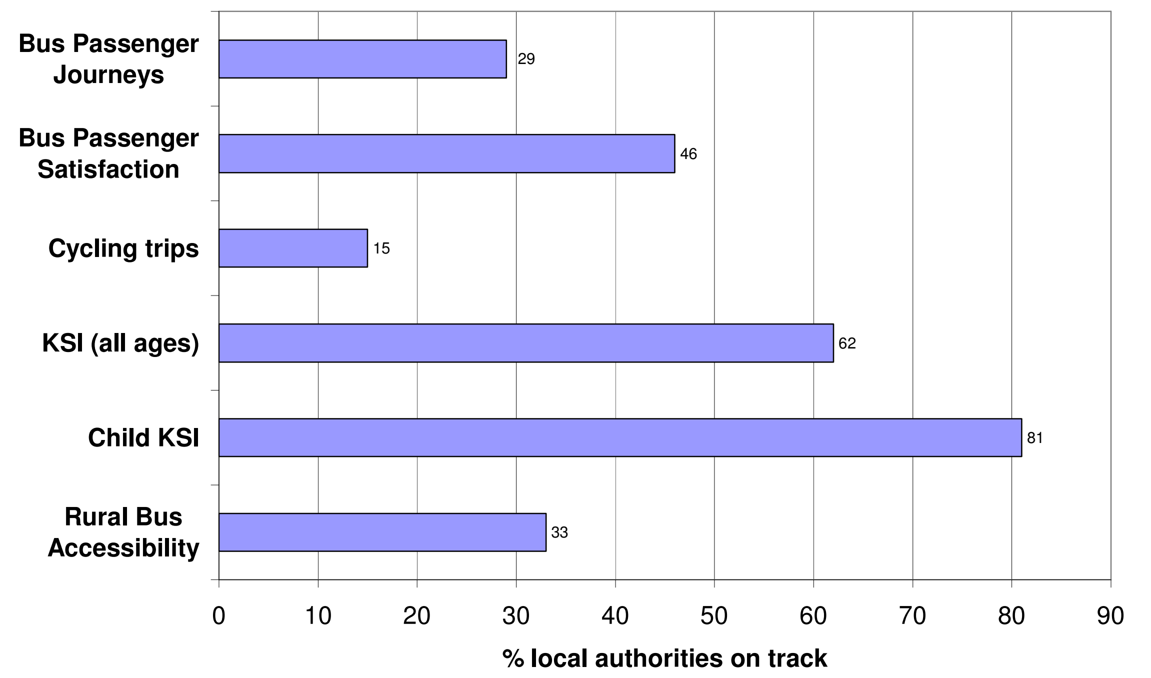
2. Background

Local Transport Plans with a 5 year horizon and a 5 year financial plan were introduced to the UK in 2000. Until then, bids for capital funding had been on an annual basis – creating considerable uncertainty and unpredictability for local transport authorities[2] [Docherty, 2003].

The 5 year Local Transport Plan is built around an objectives-led transport strategy setting out "policies for the promotion of safe, integrated, efficient and economic transport in [a local] area" [DETR, 1999]. Each Plan includes a costed implementation plan, and a corresponding bid for capital funds[3].

The first round of Local Transport Plans (or LTP1) ran from April 2000 to March 2005. These included target-setting[4], an innovation for UK local transport. We illustrate this below with monitoring data from Atkins (2005) which shows the % of authorities which in 2003/4 were 'on track' to meet their 2005 targets. 'On track' typically meant on or above a linear trajectory from 2000 to the 2005 target, for each performance indicator.

Figure 1: Interim performance in relation to targets, % of authorities 'on track'



KSI – killed or seriously injured Source: Atkins (2005)

From 2004, the funding arrangements were changed to make future capital funding partly dependent on delivery performance to date. ‘Reward funding’ of up to +25% was allocated to authorities that demonstrated good progress, particularly in the delivery of schemes and achievement against targets (Fig 1). Others were penalised by funding reductions of up to -10% (DfT,2003). Some authorities were found to have been unrealistic/overambitious in their Plans, and the new emphasis on *delivery* served both to penalise those authorities and to change the balance of incentives for the future.

For the second round of Local Transport Plans (LTP2) April 2006-March 2011, these arrangements were updated (DfT, 2004a). Funding would now be determined in two stages:

- Firstly, there would be a formulaic ‘guideline funding’ based on ‘need’. Key variables in the formula were bus and light rail patronage, road casualties, population and size of urban areas, air quality, car ownership and deprivation. Guideline funding allocations based on ‘need’ were published in 2005 for the whole 5 year period [DfT, 2005a,d].

Table 1: Examples of guideline funding allocations based on ‘need’, 2006-11

	2006/7 £m	2007/8 £m	2008/9 £m	2009/10 £m	2010/11 £m
North Lincolnshire	1.91	1.74	1.71	1.67	1.62
City of York	4.17	3.68	3.47	3.23	2.96
West Yorkshire	27.13	27.15	29.26	31.52	33.93

- Secondly, there would be ‘Performance Funding’ under which local transport authorities[2] would be eligible for variations, up or down, by 25% (maximum). The two key performance criteria would be:
  - quality of plan
  - delivery performance (DfT, 2005e) [5].

The performance funding system, and the planning and monitoring systems on which it rests, were described in a series of DfT publications (DfT, 2004a; DfT, 2005a,b,c,d,e).

In order to gain a better understanding of how the system has influenced local authorities’ behaviour and to fill in gaps in our knowledge of the system, we interviewed members of staff at six selected local authorities. These six included rural and urban authorities, Unitary and County authorities, with differing recent performance from Weak to Excellent. Within the authorities, we selected a set of interviewees involved in the LTP process at different levels, from the Head of LTP to – for example – Pavement Engineer. Selection of interviewees was by the method known as snowballing, which uses nominations by individual contacts to develop a network of those involved, from which representative sample is selected (Kelly et al, 2006).

We also interviewed a regional Government Office representative who provided a link between the Department for Transport and the local authorities in one region. This person was responsible for giving the authorities feedback on their draft Plans, and provided a benchmarking role within the region. We also met with the Department for Transport, in order to gain a better understanding of the incentive system from their perspective, as its designers and operators.

The information gleaned from the interviews informed the development of a postal survey addressed to all 85 LTP authorities that received responses from 32 (38%) (Kelly *et al*, 2006).

With the help of the evidence gained from these sources, we attempted to characterise the performance funding system as a game, which we refer to throughout the rest of the paper as the ‘real game’, to distinguish it from the ‘theoretical game’ which were constructing to represent its key features.

3. Characterising the game

Game theory texts give differing requirements for characterising a game. In an attempt to be reasonably comprehensive, we have set out to describe the following characteristics:

(i) Set of players

(ii) Motivations of the players

(iii) Set of actions the players can take

(iv) Payoffs

(v) Sequence

(vi) Information – who knows what, and when.

(i) Set of players

The players of the game can be summarised as:

- Department for Transport
- 85 * Local Transport Plan authorities.

The Department for Transport is the central government department charged with transport policy and the associated spending.

The Local Transport Plan authorities include not only a large number of individual local authorities, but also some coalitions of local authorities, who come together specifically for the purpose of local transport planning – for example, 10 authorities in Greater Manchester together produce the Greater Manchester LTP, and five in West Yorkshire produce the West Yorkshire LTP, in each case with the participation of the Passenger Transport Executive (or 'PTE') for that conurbation. In this paper, we refer to these as if they were a single authority. Hence there are 85 Local Transport Plan authorities, 85 LTP documents and 85 LTP funding allocations.

These Local Transport Plan authorities cover most of England, but not Wales, Scotland or Northern Ireland, which have their own separate planning systems. London (in fact all of Greater London) is also excluded from the English LTP system, again having a separate system of its own.

Although not listed in the set of players above, the regional Government Offices (GOs) and Regional Assemblies (RAs) are players in the real game. They are expected to work with local authorities in developing their LTPs. Their roles include ensuring consistency with the Regional Spatial Strategy and other aspects of regional co-ordination. However, our interviews suggest they play a subtle role in practice, and certainly lack the power to determine assessment criteria (DfT's role) or set targets and manage expenditure (local authorities' role). Our judgement is that GOs and RAs can be omitted from the theoretical game, consistent with the aim of modelling the key variables necessary to predict outcomes (see the next Section) – while recognising this is a simplification and that reality is more complex.

(ii) Motivations of the players

In simple terms, our interviews indicated that local transport authorities' prime motivation in playing the performance funding game is to maximise funding. Of course their transport planning activities as a whole serve wider goals, and some interviewees indicated that the prestige of being graded Excellent on the LTP would be attractive in itself and may contribute to a recognition under other other schemes (e.g. the Comprehensive Performance Assessment). Nevertheless the financial motive appeared to be dominant in this game, partly as this provided greater resources to ensure that local objectives can be achieved.

For the theoretical model, therefore, the local authority players are assumed to maximise

$$Max\left[E(V_i) - c(z_i)\right] \qquad (1)$$

where $E(V_i)$ is the expected financial reward to authority i, in the form of performance funding;

$c(z_i)$ is the cost of playing the game (or the 'total cost of effort' involved).

English local authorities are generally required by law[6] to prepare an LTP. Still, a key question is how much effort or expense an authority will go to prepare and deliver a 'good' LTP rather than just the bare minimum. This will influence the variable $C(z_i)$.

We also need to consider what benefit DfT is getting from the performance funding process, and it is clear from our interviews that the overall purpose – from DfT's viewpoint – is to incentivise local authorities. In part, the purpose is to incentivise them to pursue and deliver Government goals, in relation to a set of 13-16 'mandatory indicators' which address accessibility, safety, public transport patronage/quality/satisfaction/puntuality, cycling levels, traffic and congestion, air quality and road/footway condition[7]. Here the local authority is free only to set the level of its target. This aspect of the game could be interpreted as a pure *principal-agent relationship* in which DfT incentivises its local 'agent' (which has better information and better capability to act, locally) to pursue the principal's (DfT's) goals.

There is another aspect to the game, since the local authority is allowed to set additional 'local targets' of its own devising, up to a maximum of 40 targets of all types (DfT, 2004a:23). Most authorities have adhered to this recommendation, and the evidence suggests that the weighting placed by DfT on the local targets is 50% or less. Here, the goal-setting function is devolved from central government to local government, which makes the relationship more subtle and interesting. For example, one local authority has set six local targets alongside its 16 mandatory targets (Surrey County Council, 2006). These include:

- some local targets which refine the mandatory targets, taking advantage of better local data or a stronger local focus on the problem, e.g. increase 'Bus Satisfaction among users' from 64% in 2003 to 73% in 2009, based on community survey data every 2 years (a refinement of 'bus satisfaction among the whole population' including those who never use the bus, measured every 3 years – a mandatory target);
- some local targets which extend the scope of the LTP targets, for example, 'Population who benefit from noise reducing surface' to be increased from 11% of population in 2004 to 16% in 2010 (noise does not feature among the mandatory targets).

Are the local targets completely disconnected from central government's aims and motivations? The evidence suggests not. Firstly, they tend to be within the scope of national policy – for example, noise reduction is within the 1998 Integrated Transport White Paper and the 2004 Future of Transport White Paper (DfT, 1998; DfT, 2004a). Secondly, DfT's assessment of Local Transport Plans explicitly includes their fit with national policy: "The Plan is consistent with relevant national policies [?]" (DfT, 2005e). Therefore as part of the partnership working arrangements between DfT, regions and local authorities, it seems to us that goals and targets are being aligned, and that target-setting within the LTP is playing a part in this process.

How can we summarise DfT's motivation in the LTP performance funding game? This is complex partly because there are multiple mandatory targets, partly because of the element of devolved authority in the local targets, and partly because the way in which performance will be judged by DfT has not been formalised in a quantitative/mathematical way. A common measure is needed to describe what DfT is aiming to get out of the process – even if this is a composite of other factors.

One candidate might be 'effort', since a typical goal of incentive mechanisms is to encourage players to increase their effort. Effort is an input variable, though, and somewhat difficult to observe and measure. Instead, DfT has tended to focus on 'stretch'. This is a measure of how challenging for the local authority is a particular target level that has been set (DfT, 2004a). By extension, 'stretch' can be measured across a set of mandatory targets and local targets. And it can be applied to delivery as well the Plan, in the form:

- planned stretch – amount of stretch in an LTP target level;
- achieved stretch – amount of stretch in the delivered performance.

'Stretch' is made more concrete by defining levels of stretch for eight of the mandatory indicators (DfT, 2004a, Annex C). Note that in these cases DfT is defining what *should* be stretching, and is not differentiating between types of LAs.

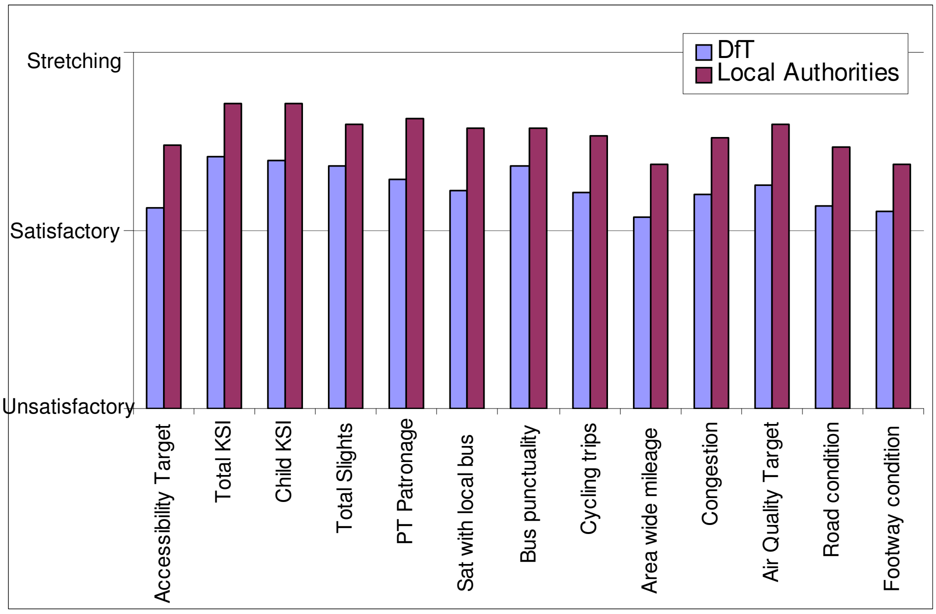
Table 2: Levels of stretch defined by DfT

Indicator	'Satisfactory' (min. standard)	'Stretching' (min. standard)
Killed/Seriously Injured (KSI)	1994-8 to 2010: -40% or 2004-10: -20%	1994-8 to 2010: -40% or 2004-10: -30%
Child KSI	1994-8 to 2010: -50% or 2004-10: -25%	1994-8 to 2010: -50% or 2004-10: -35%
Slight Casualties	No increase vs. recent levels	10% reduction
Bus punctuality	90% punctuality by 2014-5	90% punctuality by 2012-3
Bus satisfaction	Maintain to 2009/10 if >50% or increase by 6% if not	2009/10: >75% and > level in 2003/4
Mode share to school	No increase in car share to 2010	-
Cycling	No reduction	-
Urban peak traffic flows	No increase to 2010/11	-
Maintenance	No overall deterioration	-

Source: DfT (2004a). Note: '-' = undefined.

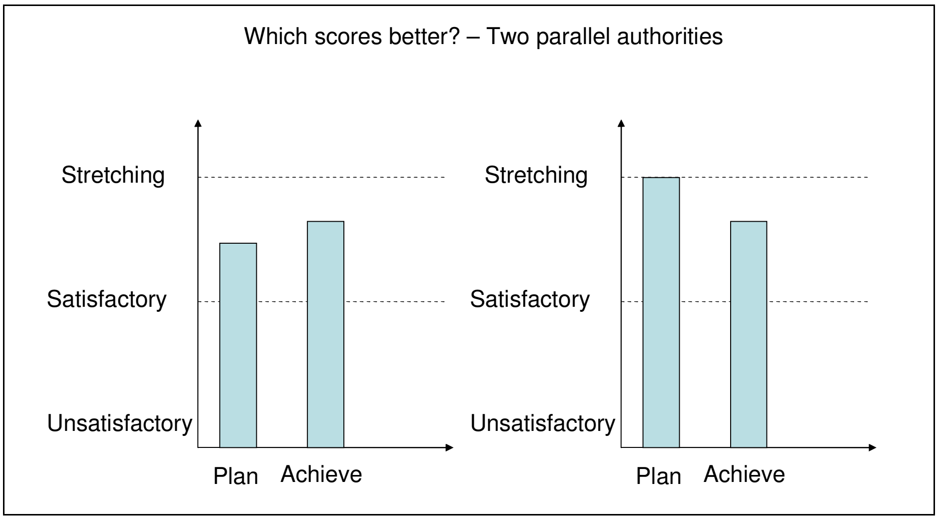
We sought to understand how local authorities' perceptions of stretch differed from DfT's assessments of stretch. At the time we did this work, DfT's assessments had not yet been made, so we compared local authoritries' own perceptions of stretch on each of their targets with their perceptions of how DfT would judge the levels of stretch on the same targets. They had the benefit of meetings with DfT and regional Government Office personnel to inform their understanding about DfT's likely judgements. Broadly speaking, we found that local authorities systematically judged their own stretch slightly higher than they perceived DfT would judge it, but otherwise there was a certain amount of consistency over the meaning of 'stretch'.

Figure 2: LAs' judgements of stretch vs their expectation of DfT's judgements



In our interviews with DfT, we asked questions to explore how stretch levels across a number of targets would be combined into an overall assessment, and to explore how planned stretch and achieved stretch (delivery performance) would be combined. These questions included scenario tests such as the following.

Figure 3: Question exploring how Planned stretch and Achieved stretch are combined (example)

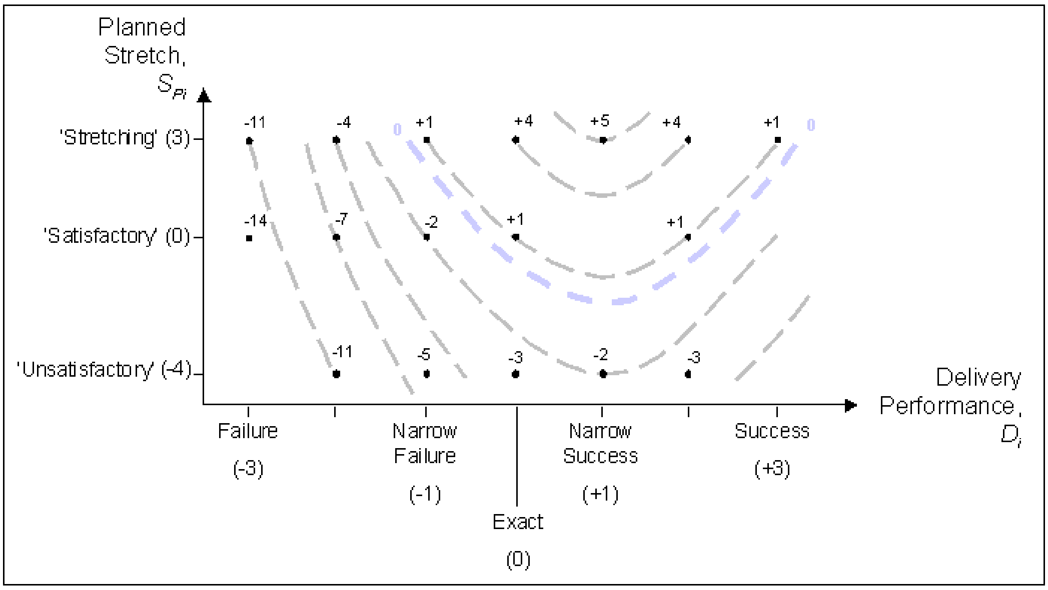


The main findings from this were that, although not all the rules are written down, we could realistically represent the game as follows:

- DfT will reward realistic and achievable rather than aspirational targets, and successful delivery of such targets – local authorities are aware of this approach;
- mandatory targets (in aggregate) and local targets (in aggregate) will be given equal weight (DfT, 2005e);
- in aggregating performance across individual targets: the overall assessment will be sensitive to any 'unsatisfactory' targets, and to any delivery performance worse than planned; in other respects, each target will be given equal weight.

Figure 4 illustrates how our interviews indicate DfT judged stretch vs delivery. On the horizontal axis, 'success' indicates the difference between achieved stretch and planned stretch. The values assigned to the axes represent our understanding of the weight applied to stretch and achievement rather than any formal values.

Figure 4: Scoring based on planned stretch versus delivery performance



This could be represented by a function such as $q_i\,(S_{Pi},D_i) = S_{Pi}+2-(D_i-1)^2$ where q_i is the score of authority i, S_{Pi}, is planned stretch, and D_i is the difference between planned stretch S_{Pi} and achieved stretch S_{Ai}. A Cobb-Douglas form might also be used, but would require a separate term or function to address the upward-sloping part of this relationship (i.e. the apparent judgement that excessive delivery over planned performance is indicative of poor planning/lack of ambition).

In the lead up to the 2006/7 capital settlements, DfT has gradually shifted the focus away from stretch/impact of targets: the weighting on this part of the assessment has been reduced from 30% down to roughly 10%, and it is now implicit rather than explicit in DfT's assessments[8]. This has implications for the incentives provided by targets, however 'quality of plan' and 'delivery performance' remain the two top-level criteria.

Although it is tempting too see this gradual shift in the rules of the game as a reflection of DfT's changing priorities, it may also be a way of keeping local authorities 'on their toes' – preventing them becoming too expert at playing the assessment. We return to the question of information below. Nevertheless, this means the definition of the 'real game' is shifting and harder to pin down. In developing a theoretical model of the game, we have had to concentrate of the core variables and bigger influences – the detail is not only hard to observe but constantly changing.

In summary, we found that the motivation of local authority players in the LTP funding game is primarily to maximise their net financial outcome, whilst DfT's motivation is more complex. We infer that DfT is keen to encourage 'effort' on the part of the local authorities, but since effort is not easily observable DfT has used targets (and measures of planned stretch and delivery performance) to help it create these incentives.

(iii) Set of actions the players can take

In short, each LTP authority has main two sets of tasks:

- prepare the Local Transport Plan – including setting targets for future performance;
- implement the Local Transport Plan – through projects, policies, expenditure, management, etc.

In addition, the local authories are required to submit Annual Progress Reports to DfT.

DfT's most important action within the game is to allocate a fixed block of funding between the 85 LTP authorities, annually. DfT also assesses the Plans submitted by the authorities and their Annual Progress Reports, and uses those assessments to inform the annual funding allocation (DfT, 2004a; DfT, 2005e). DfT also holds occasional meetings with authorities and feeds back information to them via regional Government Offices – for example, about the number of targets required and about the way in which the Plan and APR will be assessed, although the full information on the latter has tended to be held back until after the assessment.

We were interested in a number of questions about the set of actions:

- Harbring and Irlenbusch (2003) had identified collusion against the principal in some experimental contests. We explored the possibility of collusion between LTP players, but found that the large number of players made this difficult (costly to communicate and costly to commit all players), and neither DfT nor the local authorities could identify any instances in the real game.
- Holmström and Milgrom (1991) had noted that in any contest – and in particular a game focused on a narrow set of performance indicators – there might be scope for 'actions outside the game' which are of no benefit to the principal. In this case, we found that LTP authorities may take actions which would be judged favourably in other prestige competitions (e.g. Centre of Excellence status and the Comprehensive Performance Assessment) and in other transport funding streams (e.g. PIPs; the Transport Innovation Fund). However, interviewees told us that the LTP is important enough that it is regarded as a game in its own right, and that these outside influences would generally not affect the LTP players' actions.
- We were interested in LTP players' freedom of action. We learned through the interviews that there were specific limitations on this, particularly due to National Rail services being outside the control of local authorities, and due to bus operators being under pressure from Traffic Commissioners' targets as well as their own commercial goals – sometimes in

conflict with LTP goals. Whilst these are important constraints, they are common across the LTP authorities – London, which has greater powers, is not in this group.

(iv) Payoffs

The financial payoffs to the LTP authorities will be variation in their Integrated Transport Block funding, in the range ±25%. Setting the payoff for each authority could be viewed as some sort of continuous optimisation process, or the allocations could be represented by a discrete variable, e.g. +25%, +17.5%, +12.5%, +5%, nil, -5%, -12.5%, -17.5%, -25%. The 2006/7 capital settlement used discrete amounts, and capped the penalties at -12.5%, although this will revert to ±25% in future. The payoff to DfT, aside from the financial loss, is the gain in stretch or effort on the part of the local authorities – discussed under Motivation above.

(v) Sequence

The sequence of the real game over time is unusual (from a game theory perspective) in that the player plans their action first – and has that assessed by the principal – before going on to implement their action and be assessed by the principal on that as well. Furthermore, the plans are made once every five years, whilst implementation (and prizes) are enacted every year[9].

In the longer term, we would like to model this as a (complex) repeated game, however it will be very challenging to find an analytical solution to such a game, and alternative methods will be proabably be needed – e.g. simulation. The likehood that rules of the game will change again in the next five year period, 2011-16, increases the complexity.

(vi) Information

Notably, our survey found that only 16% of authorities stated they were clear as to how they would be assessed, prior to submission of the final LTP2 plans (Kelly *et al*, 2006). This reinforces the idea that the rules of the game are not tightly defined, but are open to interpretation – perhaps intentionally on the part of the principal. Despite this fuzziness of comprehension of the assessment there was a surprising degree of common acceptance of the overall objectives of maximising achievement across all targets.

4. Game theoretic model

Model type

A wide range of game theoretic models were considered, before settling on two closely-related model types, the *rent seeking contest* (Tullock, 1980) and the *rank order tournament* (Lazear and Rosen, 1981). These share the following characteristics, which match the LTP game:

- one player ('the principal') manages the game and distributes rewards;
- other players compete for these rewards;
- the sum of the rewards is fixed – allocation of the rewards is a zero sum game;
- the principal has his/her/their own motivation, and uses the game to incentivise the other players to behave in a way consistent with that;
- to compete in the game requires effort, and success is determined by a combination of effort, (potentially) ability and luck or error.

Rent seeking contests have previously been used to model political behaviour, R&D races and sporting competitions (e.g. Blavatskyy, 2004). Rank-order tournaments originated in labour economics, where they have been used to model the allocation of higher- or lower-salaried posts within a firm. The analogy here is: the firm=DfT, the employees=the local authorities and the salaries=Integrated Transport block funding allocations. To save space, we will use the rent-seeking contest here, indicating where the two models diverge.

Rent seeking contest

In a simple rent seeking contest with two players, *i* and *j*, and two prizes – one high (e.g. +25% in the LTP game) and one low (e.g. -25% in the LTP game), V_H and V_L, the game can be set up as follows.

Player *i*'s *probability of winning* over player *j* is given by the ratio of *i*'s effort to total effort

$$p_i = \frac{z_i}{z_i + z_j} \qquad (2)$$

…so if *i*'s and *j*'s efforts are equal, then *i*'s probability to win $= \frac{\alpha}{\alpha + \alpha} = \frac{1}{2}$.

The model has a random element, which becomes clearer when *i* doubles her effort but the probability to win increases to only $\frac{2\alpha}{2\alpha + \alpha} = \frac{2}{3}$.

The model can be made more deterministic by applying a higher power to the effort terms in (2), e.g. z_i^2, z_j^2. In this case, *i*'s probability to win becomes 4/5. However, in practice most rent-seeking contest models use power 1.

In order to model asymmetric ability between the players, a weight can be applied, e.g.

$$p_i = \frac{1.5z_i}{1.5z_i + z_j} \qquad (3)$$

then for equal efforts, $p_i = 3/5$.

Player *i*'s *expected payoff* is given by

$$E(\pi_i) = p_i(V_H - V_L) + V_L - c(z_i) \qquad (4)$$

where $c(z_i)$ is the cost of effort.

Therefore, given knowledge of the players and the prize(s), and given assumptions about the players' relative ability and the power function, we can identify the winning probabilities for each player, and a predicted rank order. Combined with knowledge of the cost of effort, we could predict the net payoff to each player[10].

Subjective assessment

The capital settlement documents (DfT,2005e) showed that subjective assessment was widespread in the Quality of Plan assessment. As described above, the local authorities were also not fully informed about how their plans and delivery would be assessed. We identifed a literature on subjective assessment (e.g. Baker, Gibbons and Murphy, 1994) which contains interesting findings about the role and efffectiveness of these sorts of assessments. In particular, subjective assessments can be more effective than narrow quantitative measures in capturing overall performance, but they are vulnerable to manipulation by the principal. When used together, formal assessments with observable, verifiable criteria, and subjective assessments, can provide better incentives than either type alone. We considered ways of incorporating this into our model, and chose initially to let the error terms represent the uncertainty (between the effort input and the assessed performance). It is conceivable that more- or less-subjective assessment could affect how deterministic the game is, and hence affect the equilibrium effort input by the players (see below). This would be a suitable topic for further research.

Solving the rent seeking contest with N players and one prize

In order to get closer to the real game, we need to increase the number of players and prizes. We begin with an *N* player, single prize contest (analysed by Pérez-Castrillio and Verdier, 1992).

The *contest success function* (probability to win) for player *i* is

$$p_i = \frac{z_i^r}{z_i^r + \sum_{j=1}^{N-1} z_j^r} \tag{5}$$

Hence *expected payoff* (i.e. reward net of effort) for player *i* is

$$E(\pi_i) = \frac{z_i^r}{z_i^r + \sum_{j=1}^{N-1} z_j^r} V - z_i \tag{6}$$

where z_i is the effort exerted by player *i* (assuming unit cost of effort =1)

N is the number of players

j is simply an index for all players other than *i,* and

r is a power function – the higher the power, the more deterministic the game.

In the model, it is assumed that all players including *i* set out to *maximise their expected payoff*. Substituting *B* for the "strategic environment" term, $\sum_{j=1}^{N-1} z_j^r$, which is the sum of other players' efforts *raised to the power of r,* the maximisation problem is

$$\underset{z_i}{Max}\left(E(\pi_i)\right)$$

where $E(\pi_i) = \frac{z_i^r}{z_i^r + B} V - z_i$ (7)

The first order condition for a maximum (correcting[11] Pérez-Castrillio and Verdier, 1992) is

$$\frac{d\left(E(\pi_i)\right)}{dz_i} = \frac{V z_i^r r}{z_i\left(z_i^r + B\right)} + \frac{V\left(z_i^r\right)^2 r}{z_i\left(z_i^r + B\right)^2} - 1$$

and at the maximum $\frac{d\left(E(\pi_i)\right)}{dz_i} = 0$.

Rearranging,

$$z_i^{1-r}\left(z_i^r + B\right)^2 = rBV \tag{8}$$

Whilst (8) describes all maxima, we are particularly interested in the symmetric (in effort) pure strategies Nash equilibrium, characterised by $z=z_i=z_j$

$$z^{1-r}\left(z^r + (N-1)z^r\right)^2 = rBV$$

$$z = \frac{z^r r(N-1) z^r V}{\left(N z^r\right)^2}$$

$$z = \frac{(N-1)}{N^2} rV \qquad (9)$$

At the maximum, effort z_i is

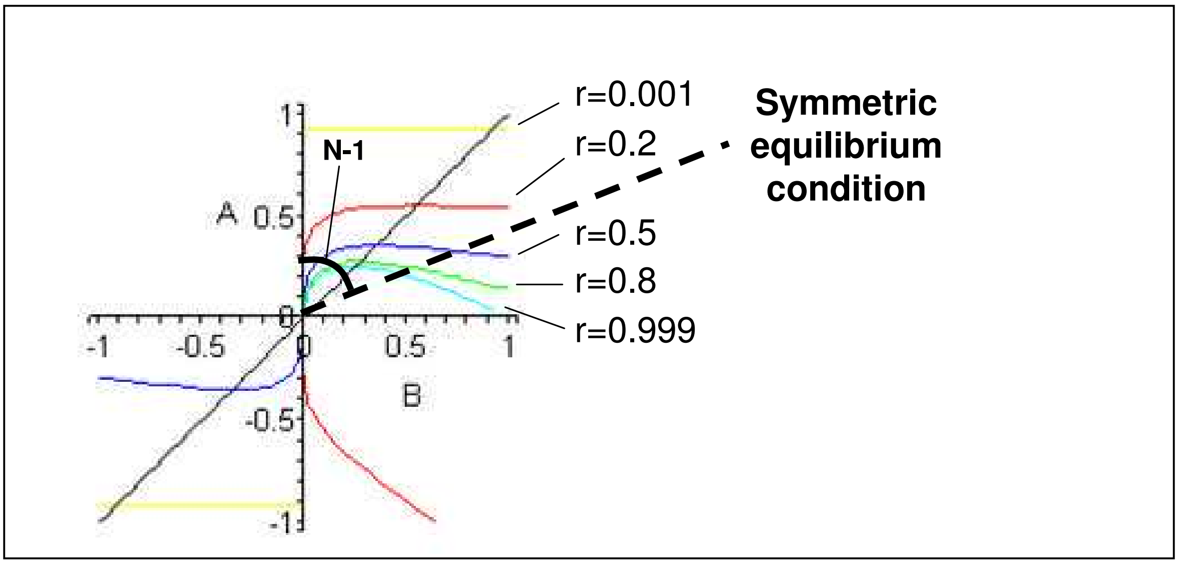
- strictly positive when $r \leq 1$ and B>0 (other players' efforts are positive in total, after applying the power function, *r*)
- strictly positive when *r*>1 provided that $0 < B \leq \frac{(r-1)^{r-1}}{r^r} V^r$
- equal to zero if *r*>1 and $B > \frac{(r-1)^{r-1}}{r^r} V^r$
- equal to zero if *r*>1 and *B*=0.

That is, whether or not it is worth player *i* actively playing the game by putting in some positive effort, depends on the value of *r* (i.e. how deterministic the game is), and on the total effort exerted by the other players (being less than a threshold value if *r*>1).

Figure 5 shows how player *i*'s optimum effort varies in the competitive environment with other players' choice of effort. If *z(B)* is player *i*'s effort given 'strategic environment' *B*, then let $A(B)=z(B)^r$, simply *i*'s optimum effort raised to the power *r*. Recall *B* is the sum of other players' efforts raised to the power *r*. The function *A* increases rapidly at first while competition is weak (B is low) but at the precise point where *A*'s effort equals the sum of others' effort, it peaks, and then starts decreasing. The symmetric Nash equilibrium is the point at which all players' reaction curves intersect in multi-dimensional space, shown by the intersection with the dashed line.

Note that as *r* increases, i.e. the more deterministic is the model, the curves in Figure 5 shift down, however $z=A^{1/r}$, and as (9) shows, the equilibrium effort is actually increasing with *r*. Also, as we might expect intuitively, equilibrium effort increases in proportion to the prize, V.

Figure 5: Reaction curves – *N* players, 1 prize



Solving the rent seeking contest with 85 players and k prizes

The LTP game is characterised by multiple prizes – in fact, the number of prizes is close to the number of players (85). Therefore the model in the previous section is not sufficient if we wish to model the game as it is in reality, nor if we wish to consider other multiple-prize incentive structures. We finally develop a model of the *N* player, *k* prize contest and solve it for 85 players (drawing on Clark and Riis, 1998).

In this game there are:

k prizes (indexed 's')

N players (indexed 'i')

v_s is the value of prize s, where s=1…k

V is the total value of the prize fund, hence $V = \sum_s v_s$

Conditional probability that player i wins the s^{th} prize:

$$P_{i,s} = \frac{z_i^r}{z_i^r + \sum_{j \in L(s)} z_j^r} \quad , \quad i, j \in L(s) \quad , \quad i \notin j \tag{10}$$

where $L(s)$ is the set of players left in the game

r is the power function.

Note that equation (10) is just the generalisation of equation (5) in the single prize game to multiple prizes.

Sticking with the notation $E(\pi_i)$ for the expected payoff, the expected payoff of player i in the game as a whole is:

$$E(\pi_i) = \frac{z_i^r}{z_i^r + (N-1)z^r} v_1 \tag{11}$$

$$+ \frac{(N-1)z^r}{z_i^r + (N-1)z^r} \frac{z_i^r}{z_i^r + (N-2)z^r} v_2 + \ldots + \prod_{h=1}^{k-1} \frac{(N-h)z^r}{z_i^r + (N-h)z^r} \frac{z_i^r}{z_i^r + (n-k)z^r} v_k - z_i$$

where h is used to index the terms in the expression;

z is the (identical) effort of each of the other players (we are focussing on the symmetric pure strategies Nash equilibrium).

Once again, set $\frac{d(E(\pi_i))}{dz_i} = 0$ and set z_i=z because by assumption we are looking for the symmetrical solution in effort. It can be shown that (Nellthorp, 2006):

$$z = \frac{r}{N}\left[\sum_{s=1}^{k}\left(1 - \sum_{h=0}^{s-1}\frac{1}{N-h}\right)v_s\right] \tag{12}$$

Interpretation: in the symmetric solution equation (12) gives the effort of each player, equivalent to equation (9) in the single prize model.

The *expected profit* of each player is found by setting z_i^r=z^r in equation (11), which can be shown to give (Nellthorp, 2006):

$$E(\pi) = \frac{1}{N}\sum_{s=1}^{k} v_s - z \quad = \frac{V}{N} - z \tag{13}$$

Clark and Riis (1998: 612) show that a sufficient condition for the existence of such an equilibrium is that the power function, r, does not exceed $N/(N-1)$ = 85/84 in this case, and the number of prizes is below 0.63N (approximately) which the current prize structure probably exceeds.

Furthermore, as the number of players increases (and 85 is a large number of players in this context) the proportion of the prize fund that must be allocated to the first few prizes in order to secure a symmetrical pure strategies equilibrium increases. If such an equilibrium is not achieved then either an asymmetric equilibrium may be achieved *or* no equilibrium may be possible. Clark and Riis do not analyse those situations, and we have not attempted to here. However, note that the symmetric equilibrium has the desirable property that it maximises aggregate effort across the players.

For the 85 player game, (12) gives:

$$z = \frac{r}{85}\left[\sum_{s=1}^{k}\left(1-\sum_{h=0}^{s-1}\frac{1}{85-h}\right)v_s\right] \quad , \quad 84 \geq k \geq 1 \tag{14}$$

In order to see the implications, consider some practical prize structures. Define *prize gradient, g*

$$g = \frac{v_1}{v_k} \tag{15}$$

Assuming a linear prize structure[12] between v_1 and v_k :

$$k\left(\frac{v_1 + v_k}{2}\right) = V$$

where V is the prize fund.

Substituting (15) and rearranging:

$$v_k = \frac{2V}{k(g+1))} \tag{16}$$

$$v_1 = \frac{2Vg}{k(g+1))} \tag{17}$$

For example, suppose that there were just four equal prizes of 25, making up a prize fund of 100. Substituting g=1, k=4 and V=100 confirms that v_1=25 and v_k=25.

Alternatively, suppose there were to be 50 prizes, with a prize gradient of 50 (i.e. the 1st prize is 50 times as large as the last prize), again with a prize fund of 100. Substituting g=50, k=50 and V=100 into the equations above gives:

$$v_k = \frac{4}{51} = 0.078$$

$$v_1 = \frac{200}{51} = 3.9$$

Sensitivity of the results

What effect would these different prize structures, in terms of number of prizes and prize gradient, be expected to have on the effort exerted by the players?

In order to assess this, we could enter the values of v_1 .. v_k into equation (14) and get a numerical result. Alternatively, we can substitute the following expression for v_s into equation (14) and use that to predict the effort level for a wide range of different k and g :

$$v_s = \frac{2Vg}{k(g+1)} - \frac{(s-1)\left(\frac{2Vg}{k(g+1)} - \frac{2V}{k(g+1)}\right)}{(k-1)}$$

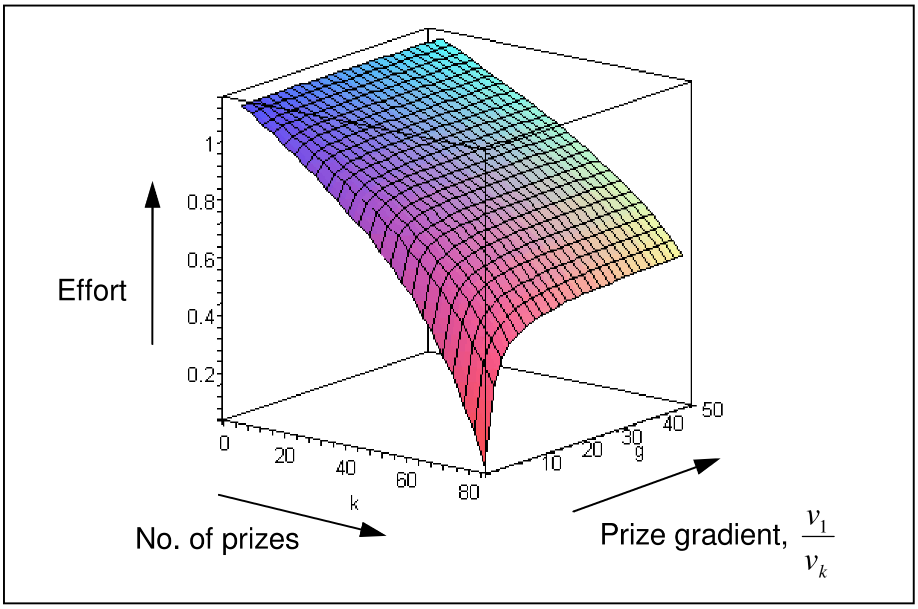
$$z = \frac{r}{85}\left[\sum_{s=1}^{k}\left(\left(1-\sum_{h=0}^{s-1}\frac{1}{85-h}\right)\left(\frac{2Vg}{k(g+1)} - \frac{(s-1)\left(\frac{2Vg}{k(g+1)} - \frac{2V}{k(g+1)}\right)}{(k-1)}\right)\right)\right] \tag{18}$$

Figure 6 shows how effort varies with the number of prizes, k, and the prize gradient, g. We assume that the prize fund V = 100, and the power function r = 1.

Figure 6 suggests that the greatest effort will be exerted when a single prize is offered. Intuitively, this is because for a given prize mass, the single prize creates the greatest competitive force between the players who, by assumption, are all equally capable of winning the prize. However, if the designer does choose to offer a large number of prizes, the prize gradient begins to play a major role. For example, when there are in excess of 60 prizes, moving from a prize gradient of 2 to a prize gradient of 10 has the effect of roughly doubling the amount of effort exerted, for the same prize fund.

Like the single prize game, (18) suggests that the effect of the power function, *r*, is to increase the players' effort for a given prize structure. Increasing the prize fund, *V*, has the effect of proportionately increasing effort. That is in line with the finding in much of the rent-seeking literature that the players will exert more effort as the aggregate rewards increase.

Figure 6: Effort responses in the 85 player game



Asymmetric players

Blavatsky (2004) and Symanski and Valletti (2004) go on to investigate the case of asymmetric ability between players, although their models are only worked out for 3 players. Their key finding is that the existence of stronger and weaker players in itself creates a case for a wider distribution of the prize mass – so that weaker players who probably cannot win the 1st prize still have an incentive to compete (invest positive effort). This creates a countervailing force to the competitive force created by the single prize. The ideal prize structure may be a 'balanced' one taking into account the incentive effects of both: i.e. the competition created by having few prizes, and the encouragement of all players to actively participate by having 'enough' prizes on offer.

5. Conclusions and further research

Incentives offered by the system overall

The LTP performance funding system can be characterised as a rent-seeking contest, in which there is a principal (DfT) and 85 local authority players competing for funding. The incentive offered to each local authority is the expectation of a financial reward *net of* the cost of effort exerted in playing the game – i.e. the effort involved in both planning and implementing a better Local Transport Plan. The financial reward is somewhat random, since the authority may do more or less well than it expects, but not wholly random since effort is linked to performance, and DfT allocates funding based on its assessment of performance. Finance was found to be local authorities' overriding motivation in playing the game.

The process of indicator selection and target-setting is used by the principal (DfT) in two ways: firstly, to align the local authorities' objectives with its own, ensuring that DfT stands to gain in kind from the authorities' efforts; and secondly (when combined with the process of LTP assessment) to calibrate the amount of stretch/ambition in the Plans and their delivery – in other words, to incentivise the authorities to be ambitious, but only within the bounds of what can fairly confidently be achieved (Figure 4)[13].

The LTP game is highly subjective, and in this type of game efficient incentives can still be provided, although this places a heavy burden on the principal (DfT). The challenge for DfT is to compensate for the lack of formal, evidence-based systems in many areas of the LTP assessment by exercising subtle subjective judgement. A benefit from this approach may be that the LTP process will avoid problems of rigidity and distortion caused by a narrow focus on a few quantitive indicators.

As an alternative, a more objective game may be easier to referee, but the achievement of a 'level playing field' (necessary for strong incentives – see below) would then rest on the design of the game (performance indicators, weighting systems, etc) to filter out the inherent ability differences between players. If successful, such a game might – the model suggests – reduce the randomness in the current game, and increase the incentive to effort without requiring an increase in the prize fund.

Details of the system

The findings point to the prize structure – how many prizes and how 'graduated' – as a key issue in the design of optimal incentives in this application. The present prize structure contains perhaps too many prizes – the model suggests that 53 prizes (=0.63*85) would be sufficient to set up a (symmetric in effort) Nash equilibrium among the players, and that further reductions in the number of prizes may further improve incentives (Figure 6). The Nash equilibrium is a desirable outcome because it incentivises the maximum aggregate effort on the part of the local authority players for a given number of prizes. All points on the surface shown in Figure 6 are Nash equilibria.

The best prize structure is also closely connected to the existence (or not) of a 'level playing field' for competition between the different local authorities – on a truly level playing field a single prize may generate the greatest competition and total effort, but with mixed ability players multiple prizes are needed, or the players may be pooled by ability[14]. We intend to examine these alternative solutions for heterogeneous players through further theoretical work. In practice, DfT has given great attention to the level playing field in the LTP game, and the fact that smaller and rural authorities have sometimes achieved the top grade for performance whilst major metropolitan authorities have sometimes been graded weak[15], suggests they may have had some success in this (DfT, 2003).

Overall, the theoretical work and evidence-gathering suggests that DfT has been pursuing a 'balanced packed of incentives' in order to modestly increase delivery performance. The clearest alternative to this emerging from the theoretical work is to offer a much smaller number of prizes, increasing the competitive force between authorities – provided the level playing field could be maintained. The exercise could, for example, be recast as a challenge fund, and the size of the few prizes greatly increased to support more ambitious Plans (as in the Transport Innovation Fund). The risk would be that although all authorities probably see themselves as potentially Fair or Good performers, there are some who doubt their ability to be in the Excellent category and for them, the incentive to exert effort and actively participate would then be lost.

Further research

The current research programme will be completed by a series of laboratory environments. The experiments will involve groups of subjects using the PLUTO software model (Bonsall, 1994) to plan and 'implement' local transport strategies in an idealised city. The software has been configured to allow the research team to play the role of the Department for Transport whilst experimental subjects simultaneously play the role of the local authorities. 8 groups, each consisting of 5 players, will each be exposed to three different experimental scenarios (no incentives, one prize and multiple prizes) to examine the impacts of incentives versus no-incentives, and multiple prizes versus a single prize, on effort and performance. In addition to testing hypotheses generated by the game theory model, this work should also allow some of the complexities of the LTP game to emerge, including the repeated nature of the game, the potential for players to learn by experience, and the potential for spirals of success or failure following funding outcomes.

Other research issues identified in this paper could be addressed in other ways. We have noted that a game with heterogeneous players will be addressed by further theoretical work. The opportunity also exists for further work on the roles of subjective assessment and the allocation of effort between multiple targets.

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Notes

1. Game theory was developed under that title by John von Neumann and Oskar Morgenstern (1947) and by John Nash (1950). Generally speaking, its aim is to formalise situations where identifable 'players' interact with one another. The outcome for each player (or 'payoff') depends not only on their own actions, but also on the actions of others. There are many formal types of game. The applications include war, natural selection and business competition, as well as games played purely for enjoyment.

2. In this paper we will use the term 'local transport authorities' or sometimes 'local authorities' as shorthand for the 'authorities responsible for the Local Transport Plans'. Due to English local government arrangements, these include a mix of Unitary Authorities (single tier local government) and County Councils (upper tier of two tier local government). The latter include the six major Metropolitan councils – Greater Manchester, Merseyside, South Yorkshire, Tyne and Wear, West Midlands and West Yorkshire – each of which has a Passenger Transport Executive co-ordinating passenger transport across the Metropolitan County area.

3. The element of capital funding we are concerned with here is the so-called Integrated Transport Block, which covers Government support for transport capital improvement schemes each costing less than £5m (DfT, 2005a). The total funding allocation for the Integrated Transport Block nationally was £547m in 2006/7 (DfT, 2005e). Funding for 'major schemes' costing >£5m, for trunk roads and inter-city rail, maintenance and revenue support, are determined separately.

4. Targets define a level of performance that the organization aims to achieve for a particular activity within a given timeframe. they usually relate back to an objective and may be expressed in terms of inputs (e.g. investment of 1bn Euro in road safety during 2003), outputs (e.g. construct 1000 new-style pedestrian crossings in 2004), or outcomes (e.g. reduce the number of fatalities by 5% per year between 2003 and 2010).

5. When first announced, there were three criteria: quality of plan; impact of targets; and delivery performance (DfT, 2004a, Annex C). These were reduced to two (quality of plan; delivery performance) by the time the first LTP2 capital settlements were announced (DfT, 2005e). Impact of targets was now only implicit in the assessment, and the weight placed on it was reduced from 30% to roughly 10%, DfT advised us.

6. The legal requirement was established by the Transport Act 2000 and the only exceptions are Greater London and authorities which were classed as Excellent under the last Comprehensive Performance Assessment – which is broader than transport (DfT, 2004b).

7. These 'mandatory indicators' derive from the so-called 'shared priority for transport' agreed between the Government and the Local Government Association in 2002 (DfT, 2004b).

8. December 2005 capital settlement letters described the latest (altered) basis for assessing the quality of plans, emphasising "Maximising value"; "The Plan will deliver the best possible results"; "improve transport outcomes" - arguably the same broad goal in different language; also giving more attention to process issues "the Plan is built on a sound analysis"; "empirical evidence used throughout"; "effectiveness of consultation" ;"robustness and quality of the process for setting and monitoring targets…includes risk assessment" (DfT, 2005e).

9. DfT have indicated to us that they may move to biennial Progress Reports in future.

10. Rank order tournaments share this payoff function, but the probability to win is formulated as $\Pr\lfloor q_j < q_i \rfloor = \Pr\lfloor z_j + \varepsilon_j < z_i + \varepsilon_i \rfloor = F(e_i - e_j)$ with q representing output, ε representing the error and F being a cumulative distribution function – fixing the shape of F gives the model a functional form. The models are very similar – they differ mainly in their treatment of the error. The rank order tournament makes it transparent that the reward is determined solely by player *i*'s place in the rank order of the players based on their performance – this is sometimes a helpful interpretation.

11. Their equation equivalent to our (8) omits the power of 2.

12. Note that the real prizes in the range *x* ±25% can also be interpreted as lower fixed amount, 0.75*x*, plus 0% to 67%. Analytically it makes no difference; the interpretation may be easier with all positive prizes.

13. In this way, the LTP system probably disincentivises risk taking by local authorities, penalising both under- and over-achievement. Since our fieldwork was completed, Moseley (2006) has investigated the targets finally set by authorities for 2006-11 on three indicators, His findings reinforce this conclusion: targets are strongly clustered at levels likely to be judged just 'satisfactory' by DfT.

14. We know from the interview work that several informal benchmarking groups exist among the players – these could form the basis for three of more distinct pools of authorities within the 85.

15. E.g. East Riding of Yorkshire (Well above average), South Yorkshire (Weak).

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